Kuwait University
Dept. of Maths. & Comp. Sc.

Math. 101
Incomplete
Final Exam.

Date : February 12, 1996.
Duration : Two Hours.
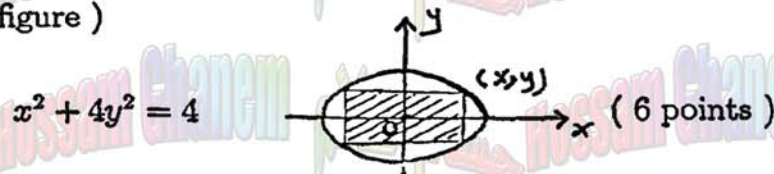
Calculators are not allowed

Answer the following questions:

1. (a) Evaluate each of the following limits, if it exists:

$(i)\ \lim\limits_{x\to\infty}\dfrac{\sin x}{x}$ $(ii)\ \lim\limits_{x\to 0}\dfrac{\cos x}{x}$ $(iii)\ \lim\limits_{x\to 1}\sqrt{x-1}$ (2+2+2 points)

(b) Find the derivative of

$$y = \tan^2\sqrt{1+\sec^2 x^3} \qquad \text{(4 points)}$$

2. (a) Find the dimensions of the rectangle of maximum area that can be inscribed inside the curve (see the figure)

$$x^2 + 4y^2 = 4$$

(6 points)

(b) Let

$$f(x) = \int_{\cos x}^{\sin x} t^3\, dt \ .$$

Find an equation for the tangent line to the graph of f at $x = \frac{\pi}{4}$.   (4 points)

3. Let

$$f(x) = \frac{x^4 + 16}{x^2}$$

(a) Find the intervals on which f is increasing or is decreasing, and find the local extrema of f (if any).

(b) Find the intervals on which the graph of f is concave upward or concave downward, and find the points of inflection (if any).

(c) Find the vertical and horizontal asymptotes for the graph of f (if any).

(d) Sketch the graph of f .   (2+2+2+2 points)

4. Evaluate the following integrals :

(a) $\int x\sin x^2\sqrt{1+\cos x^2}\,dx$.   (b) $\int_0^2 |x - \sqrt{x}|\, dx$.   (5+5 points)

5. Find the area of the region bounded by the graphs of $3y - x = 6$, $x + y = -2$ and $x + y^2 = 4$.   (6 points)

6. Find the arc length of the graph of the equation $6xy - y^4 - 3 = 0$ from $(\frac{19}{12}, 2)$ to $(\frac{14}{3}, 3)$.   (6 points)

(Good Luck)